New Physics in Mixing

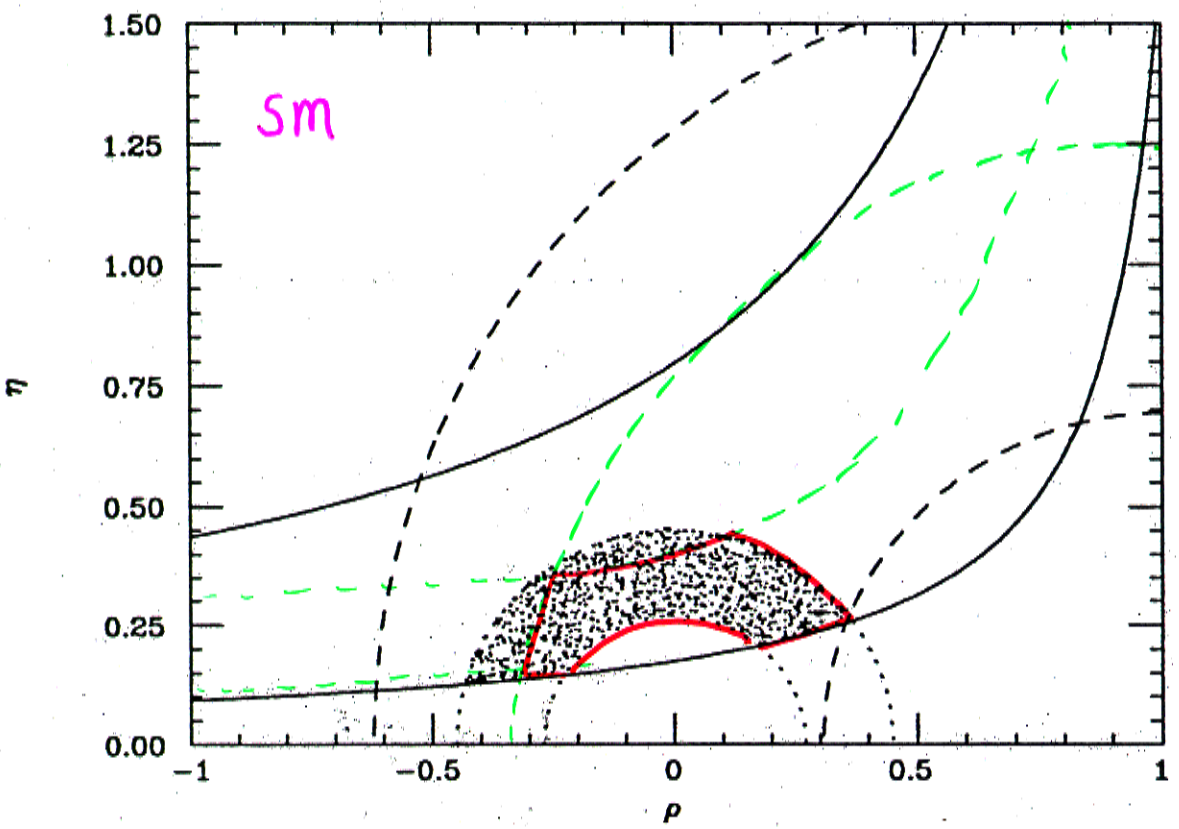
Status Report: Present Constraints
Viable + Interesting Models

- Δm_d - BaBar Physics Book
- Δm_s - Classification of Models + Effects
  Not very well studied!
- Δm_D - Charm sector potentially interesting
  Fixed target program?

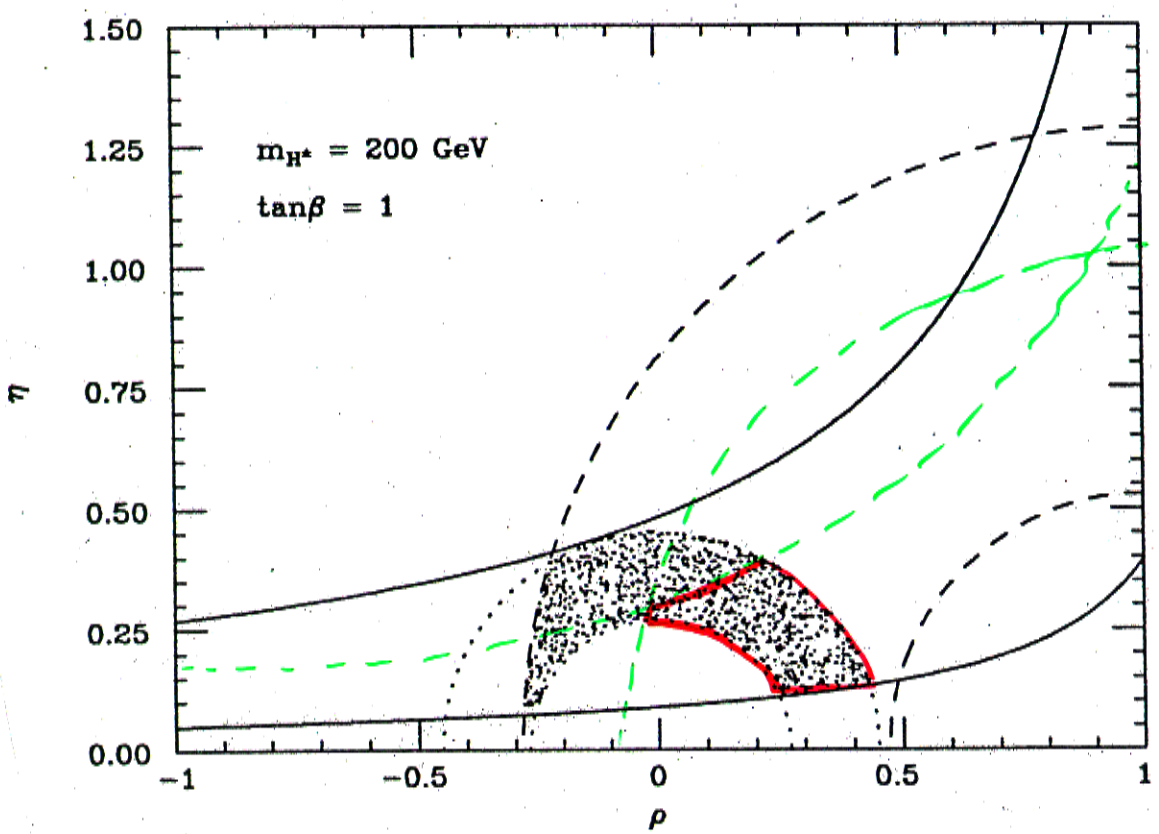
New physics in lifetime differences: see Grossman talk

Grossman hep-ph/9402348
Keum-Nierste hep-ph/9710512

Unitarity Δ - circa '94



Charged Higgs in ε, Δm_d



Can significantly affect allowed region!

Model Independent Determination of Unitary Triangle

Grossman, Nir, Worah

4 Measured Quantities:

i) $a_{\psi K_S} = \sin 2\beta$

ii) $a_{\pi\pi} = \sin 2\alpha$

iii) $\dfrac{\Gamma(b \to u \ell \nu)}{\Gamma(b \to c \ell \nu)} = \dfrac{1}{F_{ps}} \left| \dfrac{V_{cd}}{V_{ud}} \right|^2 R_u^2$, $R_u = \left| \dfrac{V_{ub}^* V_{ud}}{V_{cb}^* V_{cd}} \right| = \dfrac{\sin\beta}{\sin\alpha}$

iv) $x_d = C_t R_t^2$, $R_t = \left| \dfrac{V_{tb}^* V_{td}}{V_{cb}^* V_{cd}} \right| = \dfrac{\sin\gamma}{\sin\alpha}$

And, $\alpha + \beta + \gamma = \pi \pmod{2\pi}$

Effects of new physics in $B_d - \bar{B}_d$ mixing:

$$\frac{\langle B_d^0 | \mathcal{H}^{full} | \bar{B}_d^0 \rangle}{\langle B_d^0 | \mathcal{H}^{SM} | \bar{B}_d^0 \rangle} = (r_d e^{i\theta_d})^2$$

$(\quad + 2\theta_d)$

$(\quad - 2\theta_d)$

r_d^2

Solve (i) - (iii) for α, β, θ_d

$\rightarrow$ gives $\gamma \Rightarrow r_d$ from (iv)

Limiting Factors:

- Experimental + Theoretical Uncertainties
- 8-fold discrete ambiguity

BaBar Physics Book Study

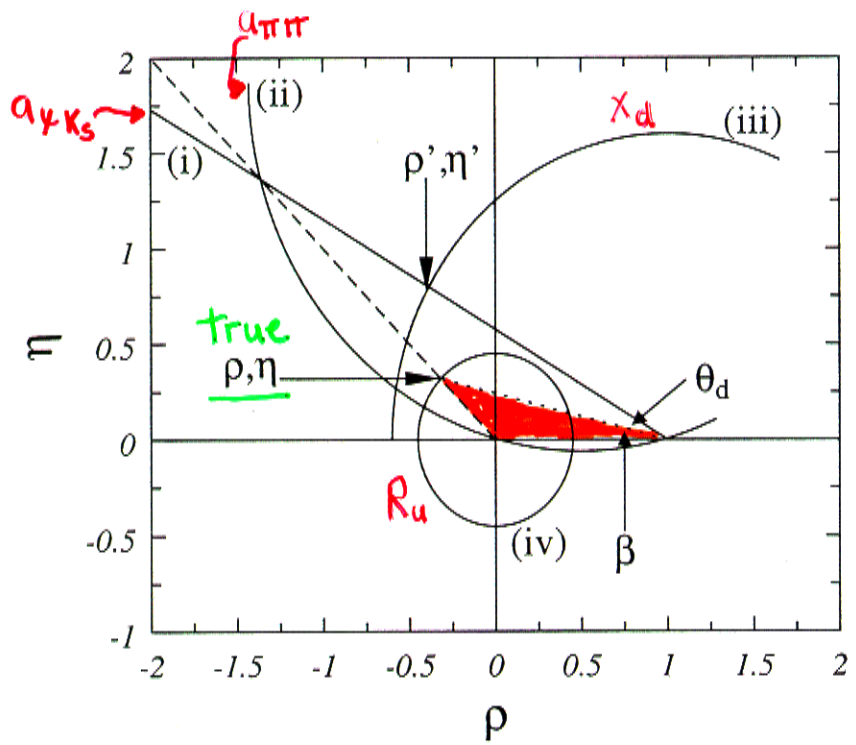
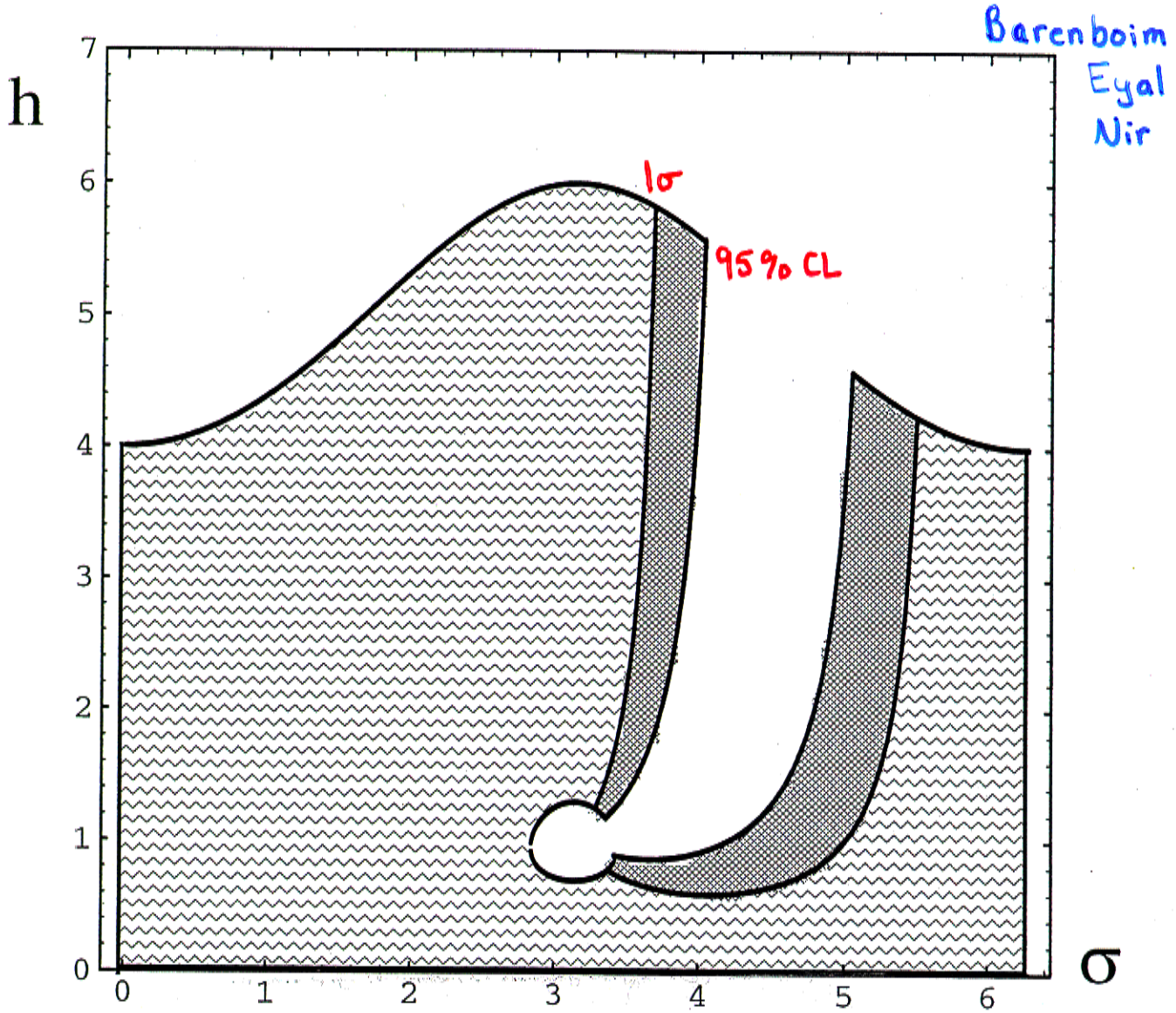


Figure 1. The model independent analysis in the $\rho - \eta$ plane: (i) The $a_{\psi K_S}$ ray; (ii) The $a_{\pi\pi}$ circle; (iii) The x_d circle; (iv) The R_u circle. The γ ray is given by the dashed line. The true β ray is given by the dotted line. Also shown are the true vertex of the unitarity triangle (ρ, η) and the (ρ', η') point that serves to find θ_d and r_d.

Parameterize: $M_{12}^{NP} = h e^{i\sigma} M_{12}^{SM}$

$$r_d^2 = [1 + 2h\cos\sigma + h^2]^{1/2}$$

$$\sin 2\theta_d = h \sin\sigma \, [1 + 2h\cos\sigma + h^2]^{-1/2}$$

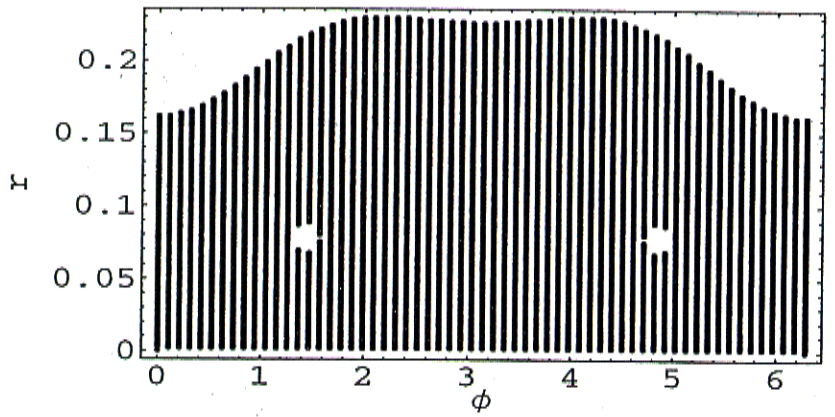


Relax 1ST assumption : 3×3 CKM is NOT unitary

Example : Singlet down-quarks - Allows flavor-changing $Z\bar{d}b$ couplings!

$$re^{i\phi} \equiv \frac{U^*_{db}}{V^*_{tb} V_{td}}$$

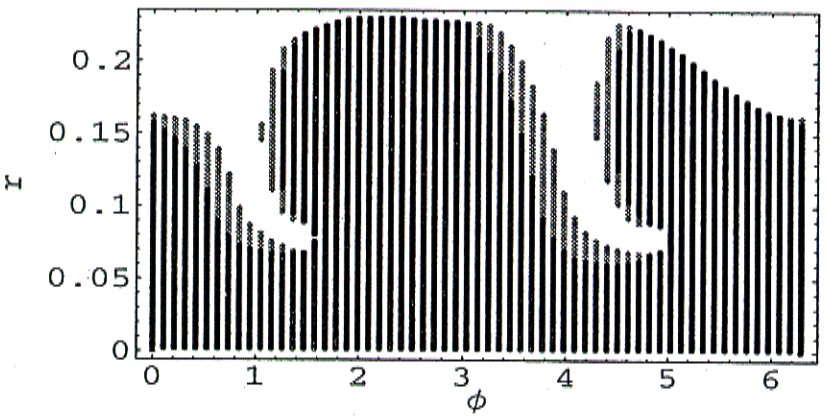
(a)



Eyal + Nir

Δm_d constraints

(b)



Δm_d + $a_{\psi K_S}$ constraints

1σ-level constraints

$$-0.8 \le \sin 2\theta_d \le 1$$

$$0.5 \le r_d \le 3.2$$

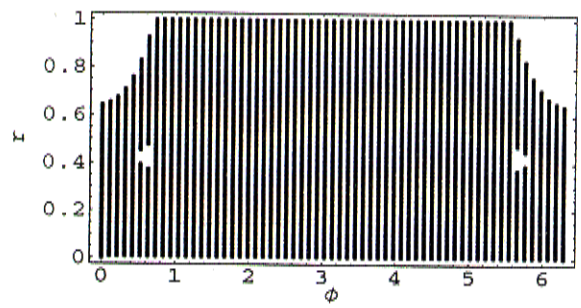
$$-4.0 \le \frac{a_{SL}}{\Gamma_{12}/M_{12}\,|_{SM}} \le 1.4$$

Example 2: Fourth Generation

$$re^{i\phi} = \frac{-V_{t'd}^{*} V_{t'b}^{*}}{V_{tb}^{*} V_{td}}$$

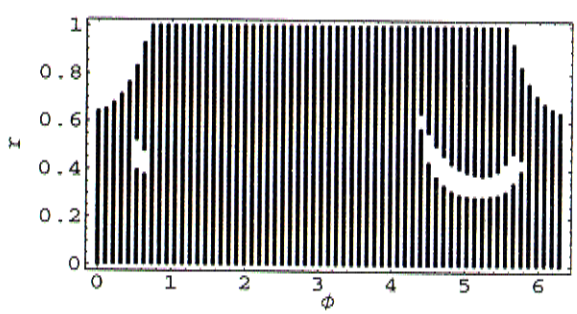
Take, $m_{t'} = 500$ GeV

(a)



Δm_d constraints

(b)



Δm_d + $a_{\psi K_S}$ constraints

1-σ constraints

$$r_d \gtrsim 0.33$$

$$-9.0 \lesssim \frac{a_{SL}}{\Gamma_{12}/m_{12}\big|_{SM}} \lesssim 6.1$$

$$-1 \leq \sin 2\theta_d \leq 1$$

Present Constraints on New Physics in B_d Mixing

Model Assumptions: 3×3 CKM is unitary

tree-level decays dominated by SM

$$\Rightarrow \Gamma_{12} \approx \Gamma_{12}^{sm}$$

Parameterize $M_{12} = r_d^2 e^{i2\theta_d} M_{12}^{sm}$

Constraints: $\Delta m_d \Rightarrow 0.3 \leq r_d^2 \leq 0.5$

CDF $a_{\psi K_s}$ + $R_u = \left| \frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*} \right| \lesssim 0.45$

gives $\sin 2\theta_d \gtrsim$ -0.6 1σ; -0.87 95% CL

[taking $|V_{ub}/V_{cb}| \leq 0.10$]

$$\frac{\Delta\Gamma_d}{\Delta m_d} = \mathrm{Re}\,\frac{\Gamma_{12}}{M_{12}} = \left.\frac{\Gamma_{12}}{M_{12}}\right|_{sm} \frac{\cos 2\theta_d}{r_d^2}$$

$$a_{SL} = \mathrm{Im}\,\frac{\Gamma_{12}}{M_{12}} = -\left.\frac{\Gamma_{12}}{M_{12}}\right|_{sm} \frac{\sin 2\theta_d}{r_d^2}$$

r_d bound $\Rightarrow$ Enhancement $\lesssim 3$

$\sin 2\theta_d$ bound $\Rightarrow$ $-3.3 \leq a_{SL} / \Gamma_{12}/M_{12}|_{sm} \leq 2.0$

Beyond the Standard Model in B_s Mixing

Present constraints : $\Delta m_s \geq 12.4\ ps^{-1}$

Take $M_{12} = r_s^2 e^{i2\theta_s} M_{12}^{SM}$

$$\Rightarrow r_s^2 \gtrsim 0.6$$

$$a_{SL} < 1.6 \times \frac{\Gamma_{12}(B_s)}{M_{12}(B_s)}$$

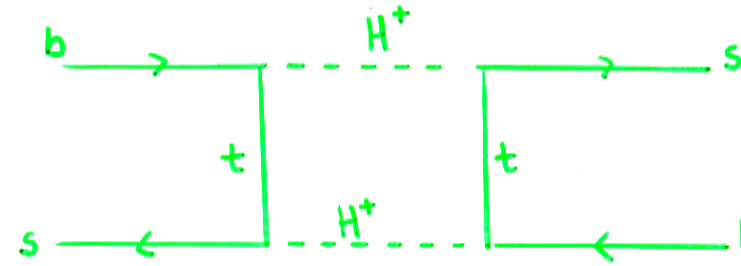
Need upper bounds on B_s CP asymmetries to constrain $2\theta_s$

$B_s \to D_s^+ D_s^-$ would be particularly useful

$$a_{D_s^+ D_s^-} \simeq \sin 2\beta_s \sim \mathcal{O}(10^{-2}) \text{ in SM}$$

Model effects in B_s Mixing

Example : Two-Higgs Doublet Model



$$\Delta m_s = \frac{G_F^2 m_W^2}{6\pi^2} f_{B_s}^2 B_{B_s} \eta_{B_s}^{QCD} m_{B_s} |V_{tb} V_{ts}^*|^2 \left[E\left(\frac{m_t^2}{m_W^2}\right) + F\left(\frac{m_t^2}{m_{H^+}^2}, \tan\beta\right)\right]$$

$$= \Delta m_d \frac{f_{B_s}^2 B_{B_s} \eta_{B_s}^{QCD} m_{B_s}}{f_{B_d}^2 B_{B_d} \eta_{B_d}^{QCD} m_{B_d}} \frac{|V_{ts}|^2}{|V_{td}|^2}$$

$\Rightarrow$ All H^+ dependence cancels in ratio !!

Holds true for most BSM

Models with non-standard CKM couplings

Fourth Generation
Singlet Quarks

SUSY – non-universal flavor couplings
$\not{R}_P$

lots of big phases

Flavor-changing Higgs
Flavor-changing Z'
Leptoquarks
Left-Right Symmetric Model – $V_R \neq V_L$

Effects in Δm_s need to be examined!

For r_s + $\sin 2\theta_s$

Including constraints from other processes.

Brhlik, Everett, Kane, King, Lebedev
hep-ph/9909480

Observable	Dominant Contribution	Flavor Content
nEDM	$\tilde{g}$, $\tilde{\chi}^+$, $\tilde{\chi}^0$	$(\delta_{dd})_{LR}$, $\sim \tilde{K}_{ud}\tilde{K}^*_{ud}$
ϵ	$\tilde{g}$	$(\delta_{ds})_{LR}$
ϵ'	$\tilde{g}$	$(\delta_{ds})_{LR}$
Δm_K	SM	SM
$K_L \to \pi\nu\bar{\nu}$	SM, $\tilde{g}$	$(\delta_{ds})_{LR}$
Δm_{B_d}	$\tilde{\chi}^+$	$\lvert\tilde{K}_{tb}\tilde{K}^*_{td}\rvert$
Δm_{B_s}	SM, $\tilde{\chi}^+$	$\lvert\tilde{K}_{tb}\tilde{K}^*_{ts}\rvert$
$\sin 2\beta$	$\tilde{\chi}^+$	$\tilde{K}_{tb}\tilde{K}^*_{td}$
$\sin 2\alpha$	$\tilde{\chi}^+$	$\tilde{K}_{tb}\tilde{K}^*_{td}$
$\sin 2\gamma$	$\tilde{\chi}^+$	$\tilde{K}_{tb}\tilde{K}^*_{ts}$
$\mathcal{A}_{CP}(b \to s\gamma)$	$\tilde{\chi}^+$	$\sim \tilde{K}_{tb}\tilde{K}^*_{ts}$
Δm_D	$\tilde{g}$	$\sim \lvert\tilde{K}_{tc}\tilde{K}^*_{tu}\rvert$
n_B/n_γ	$\tilde{\chi}^+$, $\tilde{\chi}^0$, $\tilde{t}_R$	–

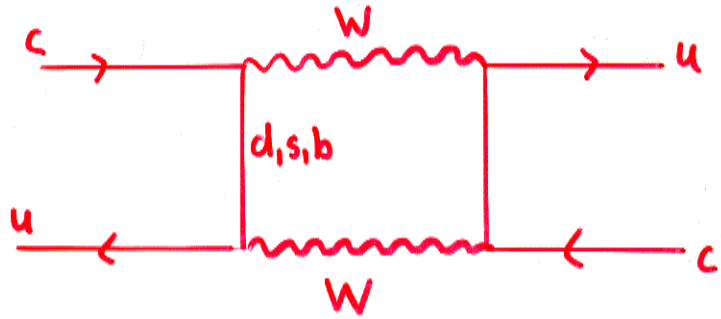
CKM + $\tilde{q}$ mass diagonalization

Table I: We list the CP-violating observables in the first column, and the dominant one-loop contributions in our framework to the amplitudes for these processes in the second column (we work in the decoupling limit and thus neglect the charged Higgs). The third column demonstrates schematically the main flavor physics. Basically the δ's are elements of the (down) squark mass matrices normalized to some common squark mass, and the $\tilde{K}$'s are super-CKM-like matrices (related to the Γ^U matrices defined in the text). Subscripts label flavor or chirality. The table is designed to demonstrate symbolically which observables are related (or not) to others. (More technically, the squark mass matrices are expressed in the super-CKM basis, in which the squarks are rotated by the same matrices which diagonalize the quarks. In the down-squark sector, we utilize the $(\delta_{ij})_{AB}$ parameters of the mass insertion approximation (in which i, j denote family indices and A, B denote helicity indices). We use a slightly non-standard but more transparent notation for the flavor indices as d, s rather than the usual $1, 2$ labels. $\tilde{K}_{ij}$ labels the flavor factors entering at the vertices of the loop diagrams involving up-type squarks. These factors are given by products of entries of the up-squark mass matrices and entries of the CKM matrix due to the quark rotations. Note also that the explicit flavor factors which enter in the $b \to s\gamma$ and the nEDM amplitudes involve right-handed quarks on the external lines, and hence are different from the $\tilde{K}$ matrices discussed below, but the flavor structure is similar (a similar statement holds for the flavor factors which enter in $D - \bar{D}$ mixing).

$D^0 - \bar{D}^0$ Mixing

$$|D_L\rangle = p|D^0\rangle + q|\bar{D}^0\rangle$$

$$|D_H\rangle = p|D^0\rangle - q|\bar{D}^0\rangle$$



Short-Distance - External momentum non-negligible

$$\mathcal{H}_{eff}^{\Delta c=2} = \frac{G_F \alpha}{\sqrt{2}\, 8\pi x_W} \left[|V_{cs} V_{us}^*|^2 \left[I_1\left(\frac{m_s^2}{m_W^2}, \frac{m_s^2}{m_c^2}\right) O_{LL} - m_c^2 I_2 O' \right] + |V_{cb} V_{ub}^*|^2 \left[I_3\left(\frac{m_b^2}{m_W^2}, \frac{m_b^2}{m_c^2}\right) O_{LL} - m_c^2 I_4 O' \right] \right]$$

$$O_{LL} = \bar{u}\gamma_\mu(1-\gamma_5)c\ \bar{u}\gamma^\mu(1-\gamma_5)c$$

$$O' = \bar{u}(1+\gamma_5)c\ \bar{u}(1+\gamma_5)c$$

$$\Delta m_D = (4.5 \pm 0.5) \times 10^{-18}\ \text{GeV}$$

CLEO - Aug '99

$\frac{x'^2}{2} < 0.05\ \%$

$x' = x\cos\delta + y\sin\delta$

Experiment: E791 - FNAL

$$r_{mix} = \frac{1}{2\Gamma^2} \left| \frac{q}{p} \right|^2 \left[(\Delta m)^2 + \frac{(\Delta\Gamma)^2}{4} \right]$$

$< 0.85\ \%$ at 90% CL $\quad [\Delta m_D \lesssim 1.5 \times 10^{-13}\ \text{GeV}]$

Long Distance Update on Δm_D

- HQET $\quad \Delta m_D \sim (1-2)\times 10^{-5}\,\Gamma$ (Georgi)

  $\simeq 10^{-17}$ GeV

  ? Reliability ? Large hadronic dynamical effects in m_c range

- 2-Particle Intermediate States (Donoghue et al)

  Use Data (!) on $D \to K^+\pi^-$

  $\Delta m_D = -(2.2 \pm 5.5)\times 10^{-16}$ GeV

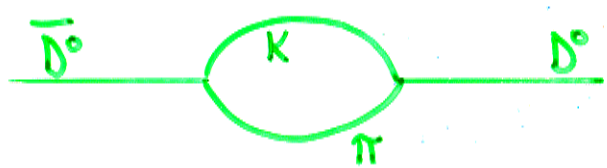
  possibility of cancellation

- Pole Amplitudes (Golowich)

  $\Delta m_D = 5\times 10^{-16}$ GeV

$$\boxed{\Delta m_{D\ L.D.} \sim 10^{-17} - 10^{-16}\ \text{GeV}}$$

Dispersive Example



$$\Delta m_D^{disp} \simeq \frac{1}{2\pi} \ln \frac{m_D^2}{\mu^2} \Big[\Gamma(D^0 \to K^+K^-) + \Gamma(D^0 \to \pi^+\pi^-) - 2\left[\Gamma(D^0 \to K^-\pi^+)\,\Gamma(D^0 \to K^+\pi^-)\right]^{1/2} \Big]$$

H. Nelson

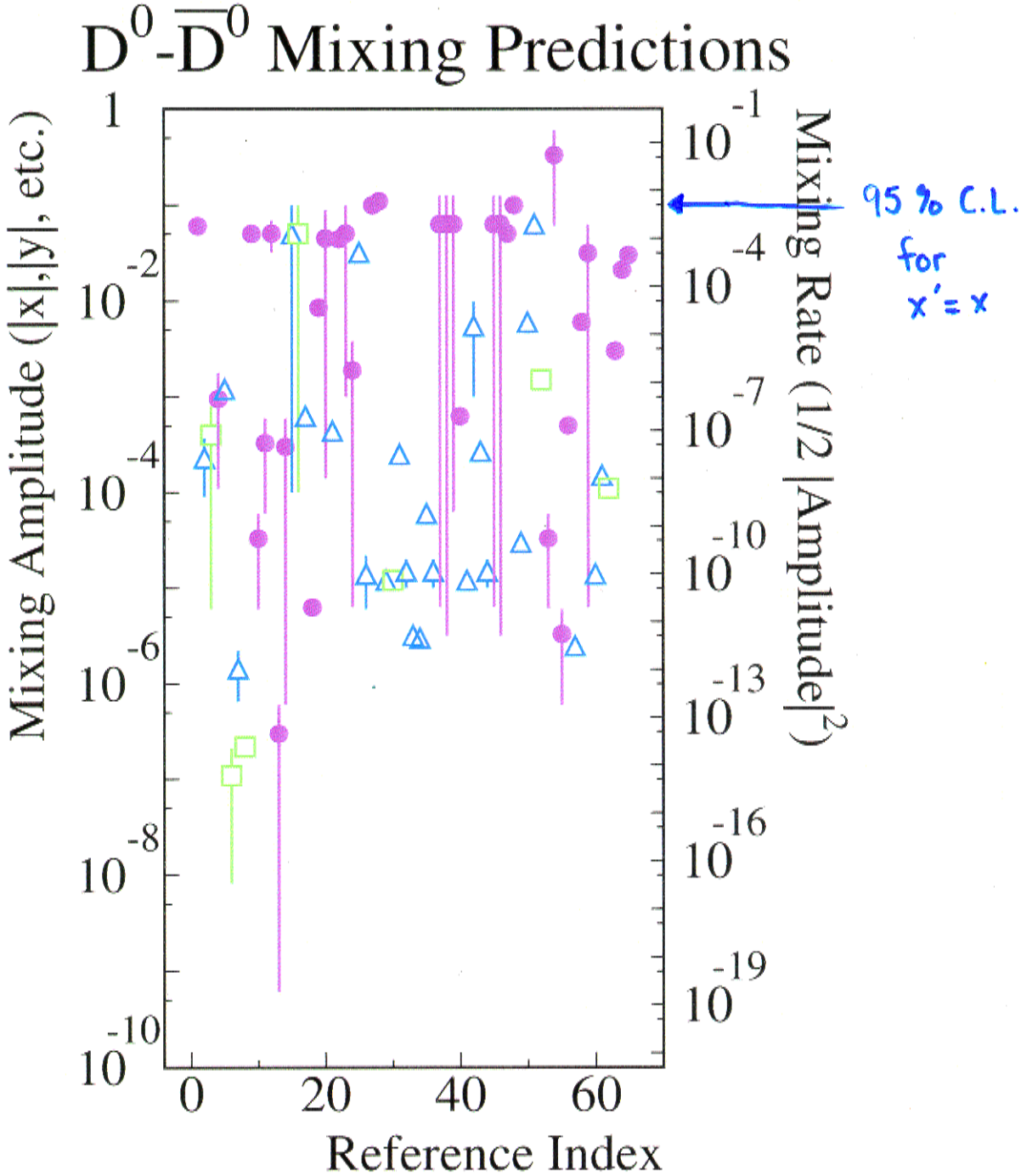


Figure 1: $D^0 - \overline{D}^0$ mixing predictions; the vertical direction, read off the left scale, is the mixing amplitude x, y, or, if the appropriate strong phase is negligible, x', y'. The right vertical scale is the equivalent mixing *rate*, which is either $(1/2)x^2$ or $(1/2)y^2$. The horizontal is the Reference Index, which is a number assigned to each prediction, and documented in Tables 1-2. The open triangles (blue) are Standard Model predictions for x, the open squares (green) are Standard Model predictions for y, and the solid circles (magenta) are non-Standard Models for x.

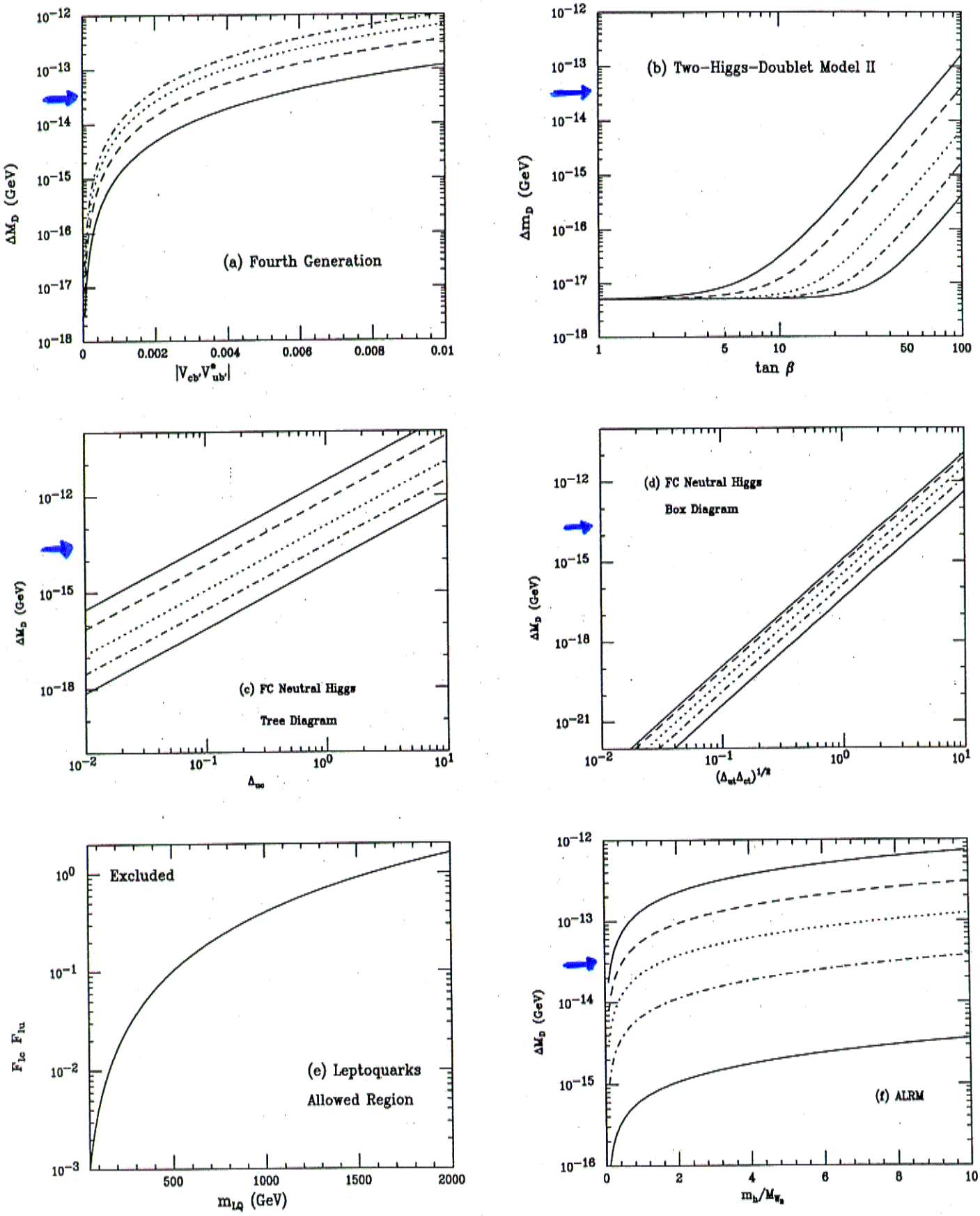


Figure 12: Δm_D in (a) the four generation SM with the same labeling as in Fig. 1, (b) in two-Higgs-doublet model II as a function of $\tan\beta$ with, from top to bottom, the solid, dashed, dotted, dash-dotted, solid curve representing $m_{H^\pm} = 50, 100, 250, 500, 1000$ GeV. The solid horizontal line corresponds to the present experimental limit. (c) Tree-

As in our approach the CP asymmetries and CKM entries are not related (since CP violation and quark mixing have different origins), it is fruitful to *define* $\sin 2\beta$ and $\sin 2\alpha$ in terms of the above asymmetries and $\sin 2\gamma$ can be defined in terms of the CP asymmetry in $B_s \to \rho K_s$; the "unitarity triangle" given in this way need not sum to 180^0 as in the SM [29,30]. As stated above, our results demonstrate that the chargino contribution alone is sufficient to account for the observed value of $\sin 2\beta$ reported in the CDF preliminary results [5]. In Fig. 1 we show contour plots of both $\sin 2\beta$ and Δm_B in the $\varphi_{\tilde{t}}$-φ_μ plane for the particular choice of parameters $\lambda' = 0.23$, and $\theta = \pi/5$.

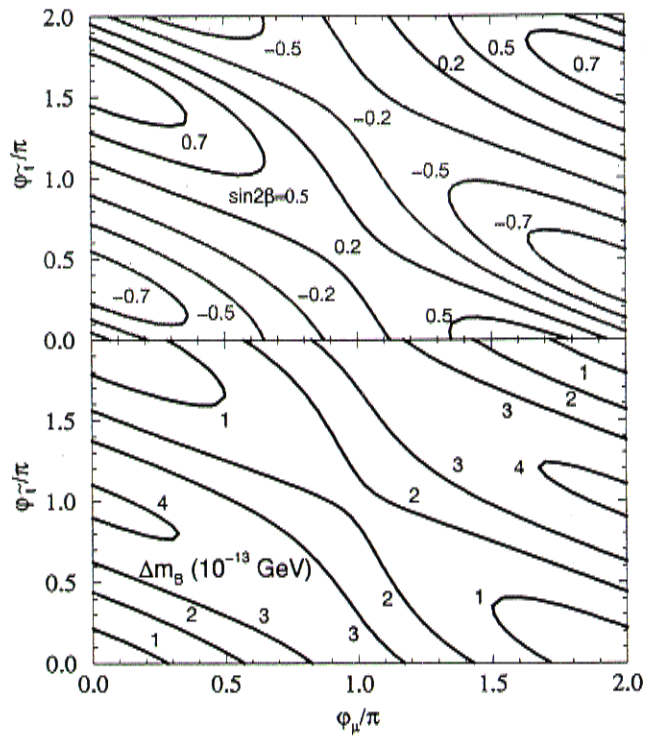


FIG. 1. Contours of $\sin 2\beta$ and Δm_B for $\lambda' = 0.23$, and $\theta = \pi/5$. In our approach both quantities originate from the SUSY contribution involving the chargino and the lightest stop; the SM contribution to Δm_B is suppressed since the CKM orthogonality condition forces $|V_{td}| \sim 0.005$, and there is no SM contribution to $\sin 2\beta$ as the CKM phase is (by assumption) zero. The absolute value of $\sin 2\beta$ can be as large as 0.78 for this choice of parameters; it can be larger for other parameter sets. Its sign depends on the sign of the stop mixing angle θ.

In addition, there is an important relation between the CP-asymmetries in $B \to \psi K_s$ and $B \to \pi^+\pi^-$:

$$\sin 2\beta = -\sin 2\alpha \; ; \tag{6}$$

this relation is a hallmark of a real CKM matrix (as pointed out previously in the context of different models by [32,33]) plus negligible direct CP violation in the B system. This relation cannot be accommodated in the SM, as can be seen using the "sin" relation:

$$\frac{\sin\beta}{\sin\alpha} = \frac{|V_{ub}|}{|V_{cb}\sin\theta_c|} \; ; \tag{7}$$