Determinations of V_{ub} from Inclusive $b \to u\bar{c}s'$ Decay

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1. Introduction and motivation.
2. Extracting V_{ub} from nonleptonic B decays.
   (a). perturbative and non-perturbative corrections
   (b). "theoretical" uncertanties
3. Conclusions and outlook.

A. Falk & A. Petrov hep-ph/9903518; J. Chay, A. Falk, M. Luke, A. Petrov hep-ph/9907363

The origin of CKM is well-known

Introduction and motivation

- SM Lagrangian Mass matrices are not diagonal in the gauge basis

$$
\begin{aligned}
-\mathcal{L}_{F,mass} &= \bar{u}'_L \mathbf{m}'_\mathbf{u} u'_R + \bar{d}'_L \mathbf{m}'_\mathbf{d} d'_R + \ \ldots \\
&= \bar{u}'_L \ S^u_L S^{u\dagger}_L \ \mathbf{m}'_\mathbf{u} \ S^u_R S^{u\dagger}_R \ u'_R + \bar{d}'_L \ S^d_L S^{d\dagger}_L \ \mathbf{m}'_\mathbf{d} \ S^d_R S^{d\dagger}_R \ d'_R + \ \ldots \\
&= \bar{u}_L \mathbf{m}_\mathbf{u} u_R + \bar{d}_L \mathbf{m}_\mathbf{d} d_R + \ \ldots
\end{aligned}
$$

- Diagonalization affects quark charged weak currents

$$
J^\mu_c = 2\bar{u}'_{L,\alpha}\gamma^\mu d'_{L,\alpha} = 2\bar{u}_{L,\alpha}\gamma^\mu \left[S^{u\dagger}_L S^d_L\right]_{\alpha\beta} d_{L,\beta}
$$

- Assume three generations $\Rightarrow$ Cabibbo-Kobayashi-Maskawa quark mixing matrix

$$
V_{CKM} = S^{u\dagger}_L S^d_L = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix},
$$

- Wolfenstein parameterization

$$
V_{CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}.
$$

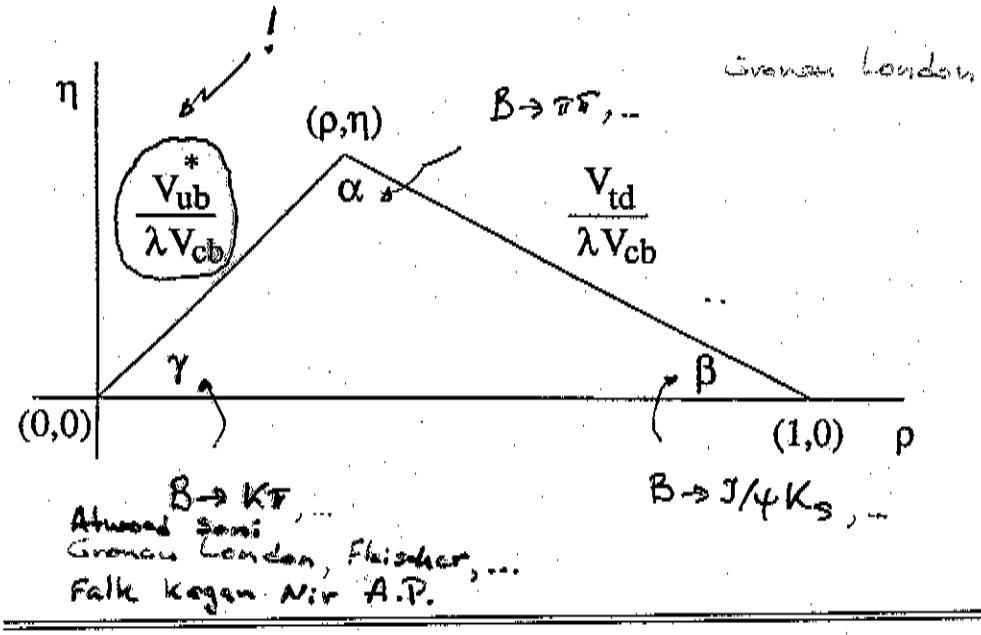
- Complex phase $\Rightarrow$ CP violation !

$\hookrightarrow \epsilon'$

- CKM matrix is Unitary by construction

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

- Unitary triangle in the complex (ρ, η) plane



$$|V_{us}| = \lambda = 0.2196 \pm 0.0023, \quad |V_{cb}| = 0.0395 \pm 0.0017,$$

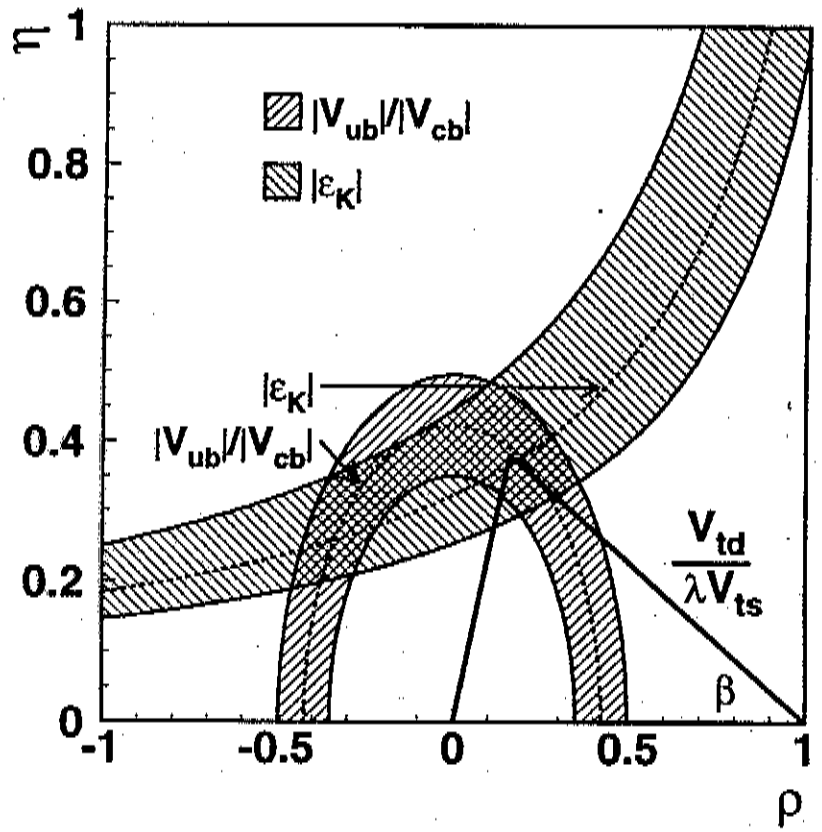
$$\underline{|V_{ub}/V_{cb}| = 0.090 \pm 0.025}$$

◇ **Overconstrain Unitary Triangle to get a grip on**

- **CP violation**
- **Possible New Physics effects**

⇒ **measure sides and angles separately**

From G. Bauer (CDF)
hep-ex/9904017

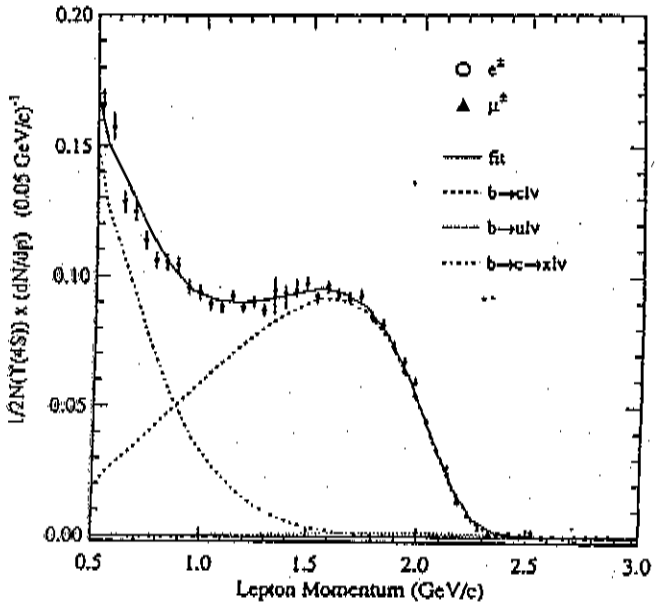


Various methods have been prop. ... excl.

Available methods for determination of V_{ub}

◇ V_{ub} from inclusive semileptonic $b \to ul\nu$ decays

Chay Georgi Grinstein Manohar Wise Blok et.al. Bigi et.al. Voloshin Shifman



- **Exp.** Problem: intractable everywhere except the endpoint
  Th. Problem: Operator Product Expansion breaks down there
  Other variables ?

Bigi et.al. Falk Ligeti Wise

◇ V_{ub} from exclusive semileptonic $B \to \pi(\rho) l\nu$ decays

Ball Braun Burdman Kambor Lellouch Ligeti Wise ...

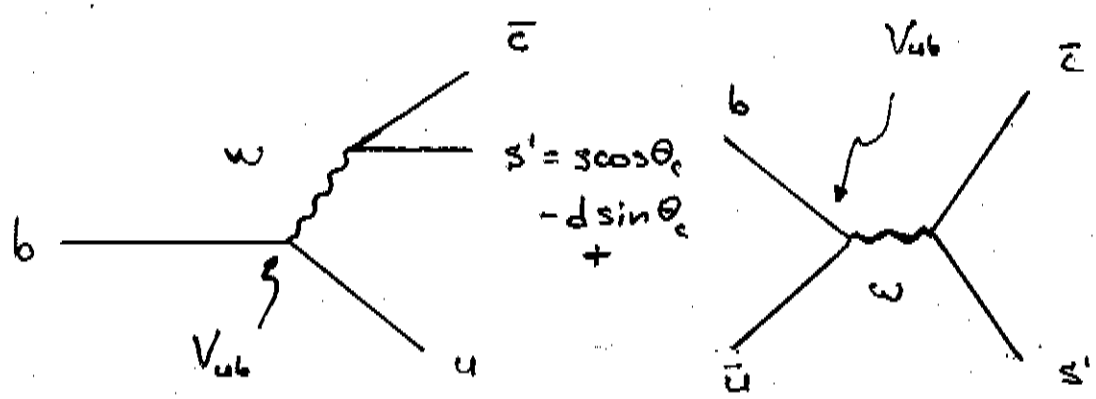
- **Exp.** Problem: tiny rate, otherwise Ok
  Th. Problem: unknown hadronization physics

  ⇒ Lattice, QCD sum rules, dispersion relations, ...

- Other methods ? Hadronic (nonleptonic) modes ?
  $b \to u\bar{u}d'$? Penguin 'pollution'...

The main problem is having $q\bar{q}$ of the same flavor. Fix it ..

Extracting V_{ub} from B decays to anti-charm



- $b \to u\bar{c}s'$ has quarks of different flavors in the final state:
  $\Rightarrow$ no contributions from penguins or final state rescatterings

A. Falk, A. P. (incl.)
B. Grinstein, R. Lebed
($B \to D^* \gamma$ excl.)
J. Chay, A. Falk, M. Luke, A. P.
M. Beneke, G. Buchalla, I. Dunietz

- Presence of $\bar{c}$ in the final state
  $\Rightarrow$ unique process signature

- Three charged particles in the final state
  $\Rightarrow$ possibly under better experimental control

- Consider ratios $\Rightarrow$ some of the theoretical uncertainties cancel

$$E.g.\quad R = \frac{\Gamma(b \to \bar{c}us')}{N_c \Gamma(b \to cl\nu)} \sim \left|\frac{V_{ub}}{V_{cb}}\right|^2$$

one of the sides of unitary triangle

- Other normalizations?

- $\Gamma_{SL}(b \to cl\nu) = \Gamma_0 |V_{cb}|^2 f(x_c)\,(1 + \ldots)$

Theoretical feasibility

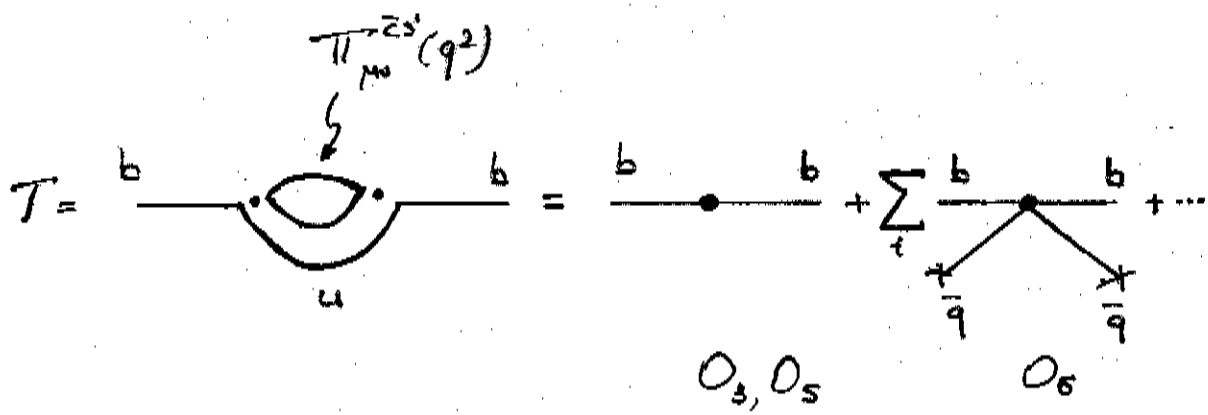
A. Falk, A. Petrov
hep-ph/9903518

- $b \to \bar{c}X$ is mediated by the $\Delta B = 1$ Lagrangian $(V-A)\times(V-A)$

$$\mathcal{L} = \frac{4G_F}{\sqrt{2}} V_{ub} \left[c_1(\mu) \bar{u}_\alpha \gamma_\mu P_L b_\alpha \, \bar{s}'_\beta \gamma^\mu P_L c_\beta + c_2(\mu) \bar{u}_\alpha \gamma_\mu P_L b_\beta \, \bar{s}'_\beta \gamma^\mu P_L c_\alpha \right] + \text{h.c.}$$

$c_1(m_b) \simeq 1.11$
$c_2(m_b) \simeq -0.24$

$$= \frac{4G_F}{\sqrt{2}} V_{ub} \left[(c_1(\mu) + \frac{1}{N_c} c_2(\mu)) \bar{u} \gamma_\mu P_L b \, \bar{s}' \gamma^\mu P_L c + 2c_2(\mu) \, \bar{u} \gamma_\mu T^a P_L b \, \bar{s}' \gamma^\mu T^a P_L c \right] + \text{h.c.},$$



- From the optical theorem

$$\begin{aligned} \Gamma(b \to \bar{c}X) &= \langle B | \mathcal{T}(b \to \bar{c}X \to b) | B \rangle \\ &= \frac{1}{m_b} \langle B | \operatorname{Im} \{ i \int d^4x \, T[\mathcal{L}^\dagger(x), \mathcal{L}(0)] \} | B \rangle \\ &\sim \Pi_{\mu\nu}^{\bar{c}s'}(q^2) \times H^{\mu\nu}(m_b). \end{aligned}$$

$\bar{c}s$ loop

- Quark-hadron duality ...

- What ratios are feasible theoretically and experimentally?

$$R_1 = \frac{\Gamma(b \to \bar{c}us')}{N_c\Gamma(b \to cl\nu)} \simeq \chi \left|\frac{V_{ub}}{V_{cb}}\right|^2 (1 + g(x_c) + ...)$$

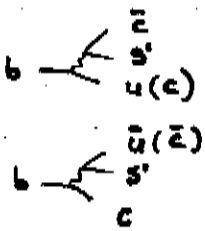
$$R_2 = \frac{\Gamma(b \to cX) - \Gamma(b \to \bar{c}X)}{\Gamma(b \to c\bar{u}d')} = 1 - \frac{\Gamma(b \to \bar{c}us')}{\Gamma(b \to c\bar{u}d')}$$

$$R_3 = \frac{\Gamma(b \to cX) - \Gamma(b \to \bar{c}X)}{N_c\Gamma(b \to cl\nu)} = \frac{\Gamma(b \to c\bar{u}d')}{N_c\Gamma(b \to cl\nu)} - R_1$$

- R_1: the simplest ratio one can form, but:

  Background from $b \to c\bar{c}s'$: *get rid of it experimentally?*

- R_2: *get rid of $b \to c\bar{c}s'$ theoretically!*



$$\Gamma(b \to \bar{c}X) = \Gamma(b \to u\bar{c}s') + \Gamma(b \to c\bar{c}s'),$$

cancel out

$$\Gamma(b \to cX) = \Gamma(b \to c\bar{u}d') + \Gamma(b \to c\bar{c}s')!$$

- R_3: different normalization, more measurements...

$$R_3 = 1 + \delta_3 - R_1, \quad \delta_3 = \frac{\Gamma(b \to c\bar{u}d')}{N_c\Gamma(b \to cl\nu)} - 1$$

- ... and a very different experimental problem !

R_1: components of the final answer

- Parameterize the 'polarization tensor'

$$\Pi^{\bar{c}s'}_{\mu\nu}(q) = \frac{N_c}{8\pi}\left[A(q^2)\left(q_\mu q_\nu - q^2 g_{\mu\nu}\right) + B(q^2) q_\mu q_\nu\right],$$

- The decay rate $b \to \bar{c} X_{ud'}$ is

$$\Gamma(b \to u\bar{c}s') = 3N_c\Gamma_0|V_{ub}|^2 x_c^4 \int_1^{1/x_c} \mathrm{d}z\,(1/x_c - z)^2 \times \left[\underline{A(z)}\,(2z + 1/x_c) + \underline{B(z)}/x_c\right],$$

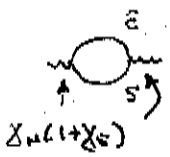
$A = A_0 + A_1 + \ldots$

$B = B_0 + B_1 + \ldots$

with $x_c = m_c^2/m_b^2$ and $\Gamma_0 = G_F^2 m_b^5/192\pi^3$.

note m_b^5!

- At the leading order



$$A_0(z) = \chi\, a_0(z) = \tfrac{2}{3}\chi\,(1 - 1/z)^2\,(1 + 1/2z), \qquad \sim \alpha_s^0$$

$$B_0(z) = \chi\, b_0(z) = \chi\,(1 - 1/z)^2/z \qquad \sim \alpha_s^0$$

$$\Gamma(b \to cl\nu) = \Gamma_0|V_{cb}|^2 f(x_c), \qquad \Gamma(b \to u\bar{c}s') = N_c\,\Gamma_0|V_{ub}|^2 f(x_c)!$$

$$f(x_c) = 1 - 8x_c + 8x_c^3 - x_c^4 - 12x_c^2 \ln x_c$$

$$\boxed{\begin{aligned} R_1 &= \Gamma(b \to \bar{c}X_{ud'})/N_c\Gamma(b \to cl\nu) \\ &= \chi\,|V_{ub}/V_{cb}|^2\,\{1 + \mathcal{O}(\alpha_s, 1/m_b^2)\} \end{aligned}}$$

$\alpha_s,\ 1/m_b^2,\ 1/m_b^3, \ldots$

$(C_1 + C_2/N_c)^2 + 2C_2^2/N_c \simeq 1.09$ ($m_b = 4.8$ GeV)

No $1/m_b$ correction:

BBSUV/ CGG Thm!

Note: no phase space factor!

R_2: components of the final answer

- Form 'asymmetry' to cancel $b \to c\bar{c}s$

$$R_2 = \frac{\Gamma(b \to cX) - \Gamma(b \to \bar{c}X)}{\Gamma(b \to c\bar{u}d')} = 1 - \frac{\Gamma(b \to \bar{c}us')}{\Gamma(b \to c\bar{u}d')}$$

$$\Gamma(b \to \bar{c}X) = \Gamma(b \to u\bar{c}s') + \underline{\Gamma(b \to c\bar{c}s')},$$

$$\Gamma(b \to cX) = \Gamma(b \to c\bar{u}s') + \underline{\Gamma(b \to c\bar{c}s')}!$$

◇ **Exp** R_2 requires separation of $b \to c\bar{u}d'$ and $b \to c\bar{c}d'$

$\Rightarrow$ easier problem!

◇ **Th.** Some of the LO QCD corrections cancel out in $\delta_2 = 1 - R_2$

$\Rightarrow$ milder scale dependence!

$$\begin{aligned} R_1 &= \left|\frac{V_{ub}}{V_{cb}}\right|^2 \{1 + h_1(x_c, \mu)\,\frac{\alpha_s}{\pi} + \ldots\}, \\ 1 - R_2 &= \left|\frac{V_{ub}}{V_{cb}}\right|^2 \{1 + h_2(x_c, \mu)\,\frac{\alpha_s}{\pi} + \ldots\} \end{aligned}$$

R_3: components of the final answer

- Form 'asymmetry' to cancel $b \to c\bar{c}s$, different normalization

$$R_3 = \frac{\Gamma(b \to cX) - \Gamma(b \to \bar{c}X)}{N_c \Gamma(b \to cl\nu)} = \frac{\Gamma(b \to c\bar{u}d')}{N_c \Gamma(b \to cl\nu)} - R_1$$

- In **ideal** world,

$$\frac{\Gamma(b \to c\bar{u}d')}{N_c \Gamma(b \to cl\nu)} = 1 \; + \; \textit{known QCD corrections}$$

$\Rightarrow$ 'Easy' measurement and 'clean' theory

- In **real world**, *'known QCD corrections'* are not known to the sufficient precision ($\sim 1\%$)...

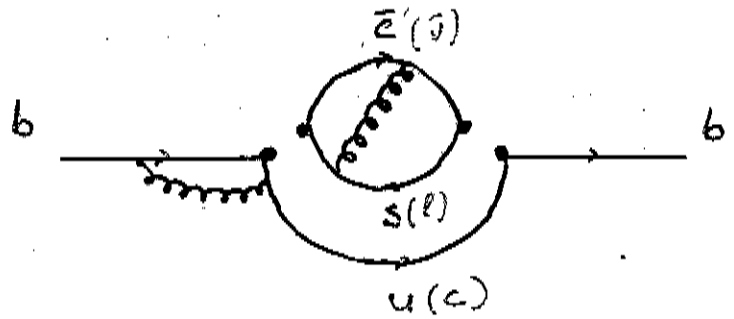
$\Rightarrow$ Measure $\delta_3 = \Gamma(b \to c\bar{u}d')/N_c\Gamma(b \to cl\nu)$ as well...

$\diamond$ Thus, at this point, R_3 is equivalent to R_2...

Perturbative QCD corrections

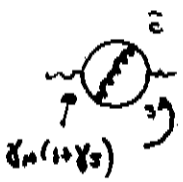
- Charm mass effects tend to enhance the ratio R_1

Singlet O_1



$b \to u \bar{c} s'$
vs.
$b \to c \ell \bar{\nu}$

- Main effect: corrections to $\Pi^{\bar{c}s'}_{\mu\nu}(q)$:



$$A_1(z) = \chi a_0(z) \frac{4\alpha_s}{3\pi} \left[f_1(z) + \frac{2z}{1+2z} f_2(z) \right], \qquad \sim \alpha_s^1$$

$$B_1(z) = \chi b_0(z) \frac{4\alpha_s}{3\pi} [f_1(z) - 1], \qquad \sim \alpha_s^1$$

- This corrects R_1 in the following way

$$R_1 = \chi \, |V_{ub}/V_{cb}|^2 \, \{1 + g(x_c) \frac{\alpha_s}{\pi} + \ldots\},$$

with $g(x_c) = a_{bu}(x_c) - a_{bc}(x_c) + a_{\bar{c}s'}(x_c)$

- Singlet (O_1) contribution only ... full NLO ?

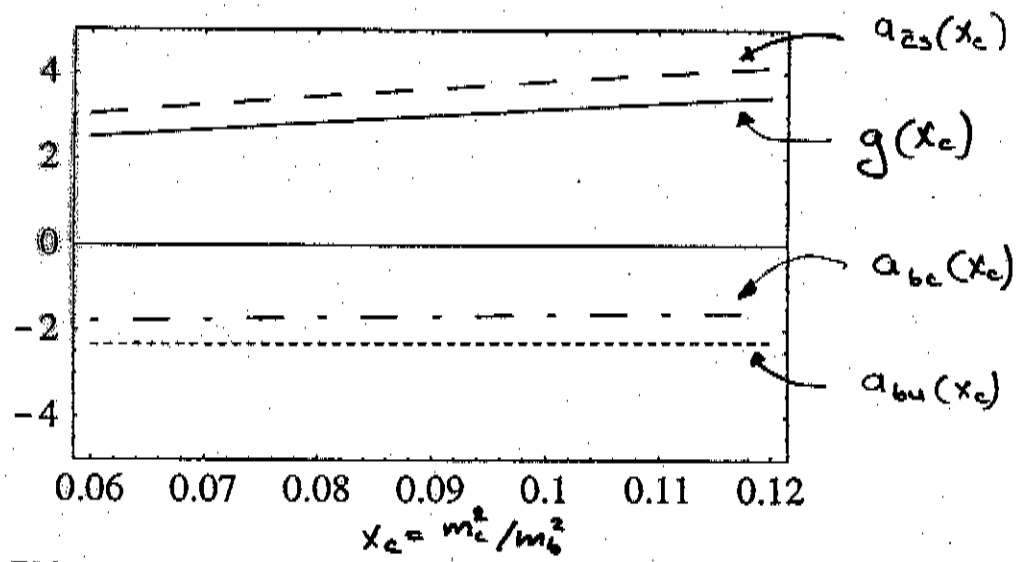
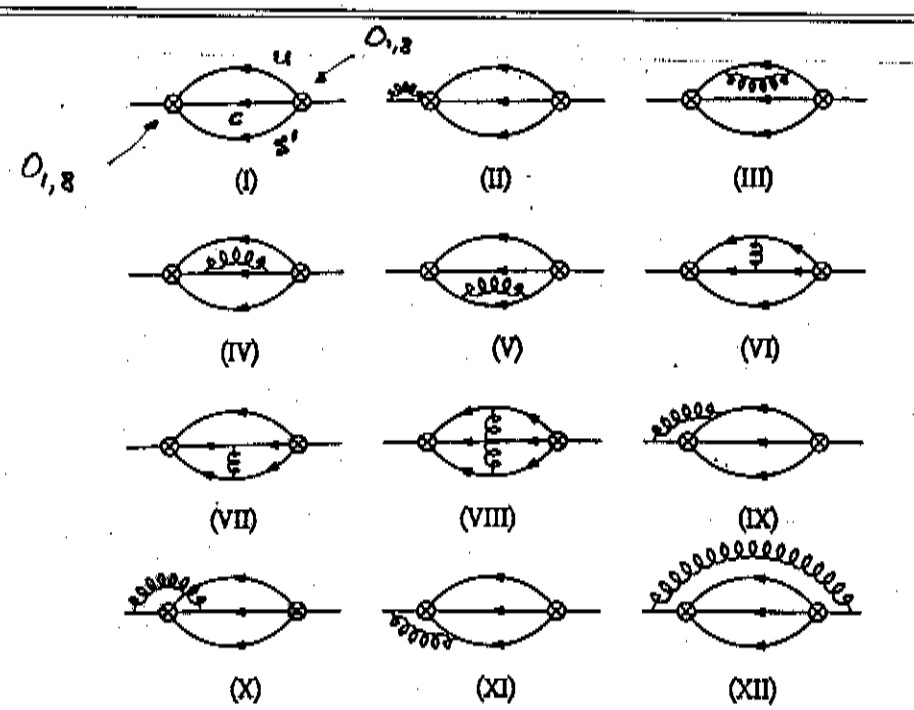


FIG. 1. Radiative corrections to R_s as a function of x_c. We display curves for $a_{\bar{c}s'}$ (dashed), a_{bu} (dotted), a_{bc} (dotted-dashed), and the total $g(x_c)$ (solid).

Perturbative QCD corrections: full NLO



$$\Gamma(b \to u\bar{c}s') = \frac{1}{3}\Gamma_0 \Big\{ 2L_+^2(\mu) + L_-^2(\mu)$$

$$+ \left[\frac{\alpha_s(m_W)}{\pi} - \frac{\alpha_s(\mu)}{\pi}\right]\left[2L_+^2(\mu)R_+ + L_-^2(\mu)R_-\right]$$

$$+ \frac{\alpha_s(\mu)}{2\pi}\left[\left[L_+(\mu) + L_-(\mu)\right]^2 c_{22}(x_i) + \left[L_+(\mu) - L_-(\mu)\right]^2 c_{11}(x_i)\right]$$

$$+ \frac{\alpha_s(\mu)}{3\pi}\left[L_+(\mu)^2 - L_-^2(\mu)\right]c_{12}(x_i)\Big\}$$

$$L_\pm = \left[\frac{\alpha_s(m_W)}{\alpha_s(\mu)}\right]^{\gamma_\pm^{(0)}/2\beta_0}, \qquad R_\pm = B_\pm + \frac{\gamma_\pm^{(0)}}{2\beta_0}\left(\frac{\gamma_\pm^{(1)}}{\gamma_\pm^{(0)}} - \frac{\beta_1}{\beta_0}\right)$$

$$B_\pm = \pm B\,\frac{N_c \mp 1}{2N_c}.$$

matching coefficients

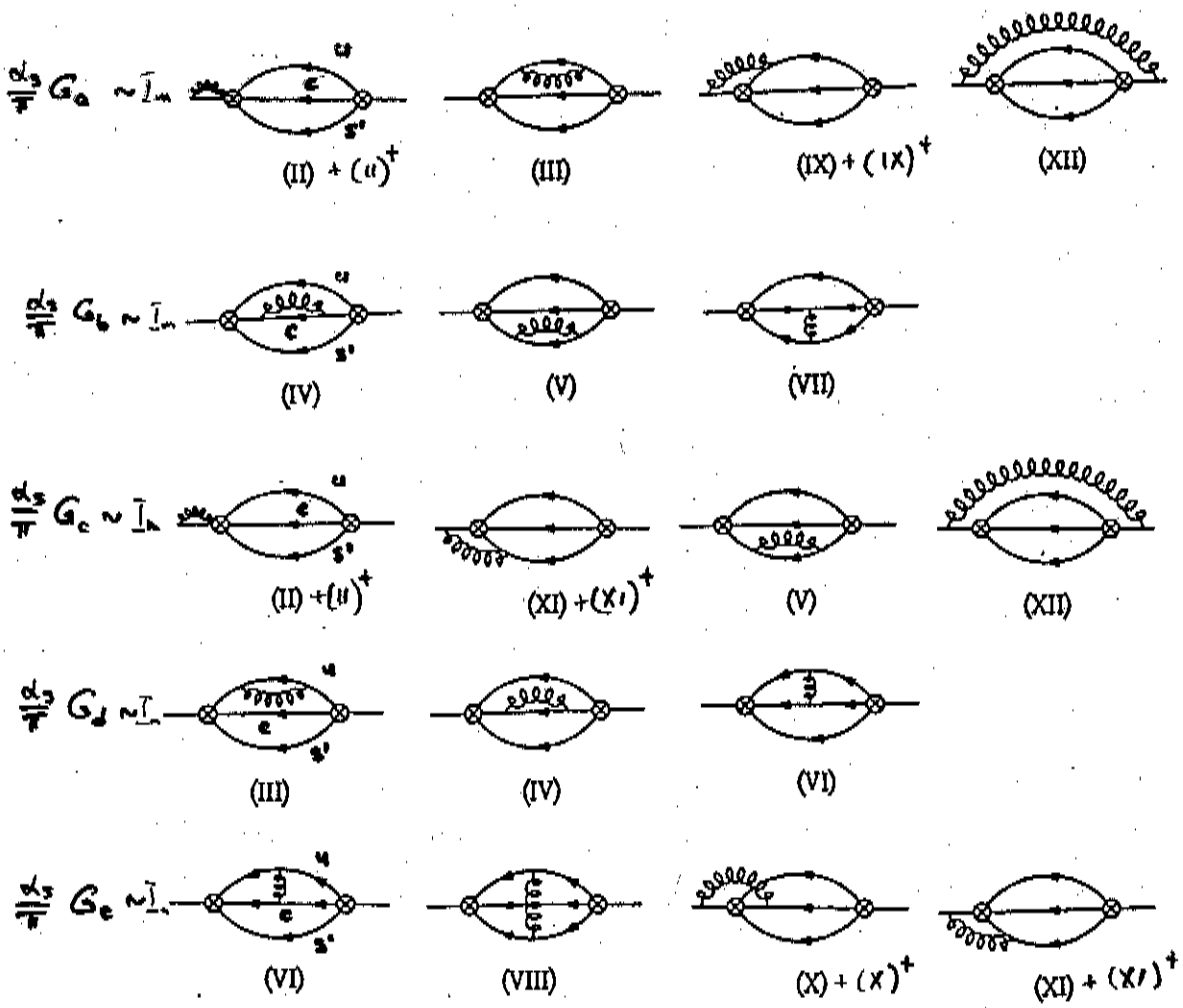
Chau, Falk, Luke, A.P.; E. Bagan et.al. Nucl. Ph. B432 (1994) 8.

- Salvage parts of the answer from other processes

$$c_{11}(\mathbf{r}_c) = G_c + G_d$$

$$c_{22}(\mathbf{r}_c) = G_a + G_b$$

$$c_{12}(\mathbf{r}_c) = G_a + G_b + G_e + B$$



$b \to u \tau \bar{\nu}_\tau$

"singlet" correction (polar.-tensor)

$b \to u \tau \bar{\nu}_\tau$

"singlet" correction (pol.-tensor)

...

$$R_1 = \mathcal{X}\left|\frac{V_{ub}}{V_{cb}}\right|^2\left(1 + h_1(x_c,\mu)\frac{\alpha_s(\mu)}{\pi} + \dots\right) \equiv \mathcal{X}\left|\frac{V_{ub}}{V_{cb}}\right|^2\left(1 + r_1(\mu, x_c)\right);$$

plot

$$\delta_2 = 1 - R_2 = \left|\frac{V_{ub}}{V_{cb}}\right|^2\left(1 + h_2(x_c,\mu)\frac{\alpha_s(\mu)}{\pi} + \dots\right) \equiv \left|\frac{V_{ub}}{V_{cb}}\right|^2\left(1 + r_2(\mu, x_c)\right);$$

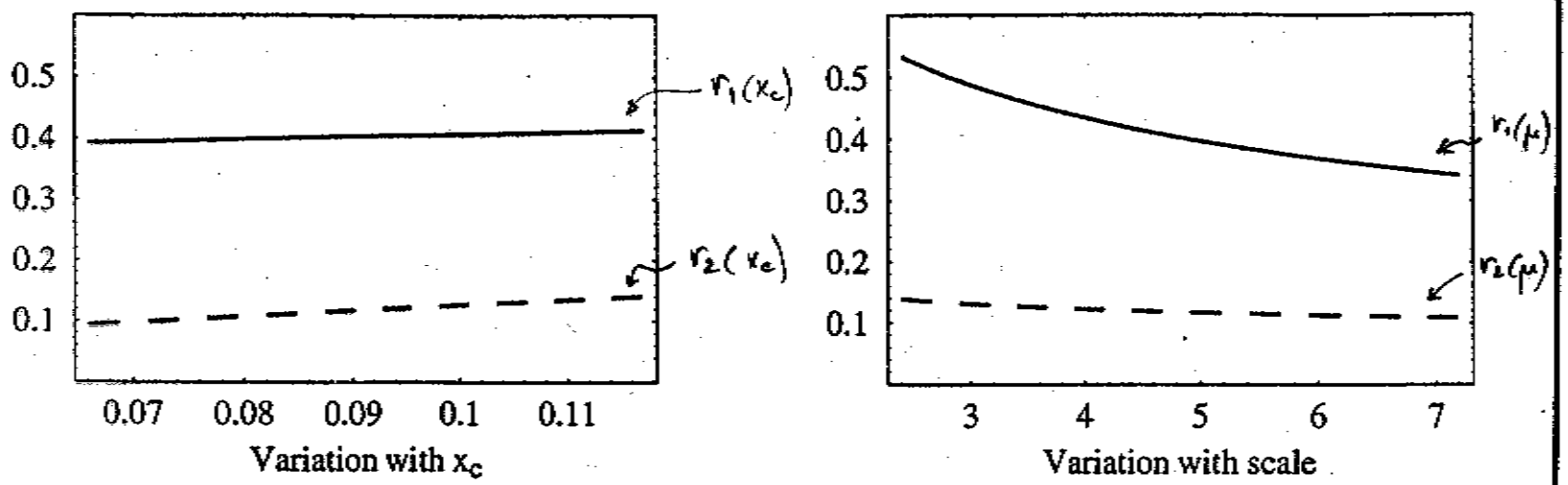


FIG. 1. Variation of r_i with x_c and μ. In both plots, the solid line is r_1 and the dashed line is r_2. (a) On the left, we plot $r_i(x_c)$ for $4.5\,\text{GeV} \le m_b \le 5.1\,\text{GeV}$ and $\mu = m_b$. (b) On the right, we plot $r_i(\mu)$ for $2.4\,\text{GeV} \le \mu \le 7.2\,\text{GeV}$ and $m_b = 4.8\,\text{GeV}$.

Perturbative QCD corrections: BLM

- Full NNLO: too hard... not expected to be large anyway
- Calculate the "BLM" part ($\sim \beta_0 \alpha_s/\pi$)

$\Rightarrow$ Singlet part: corrections factorize

$$\Gamma(b \to X) \propto 1 + a_1^X \frac{\alpha_s(m_b)}{\pi} + a_2^X \beta_0 \left(\frac{\alpha_s(m_b)}{\pi}\right)^2 + \ldots,$$

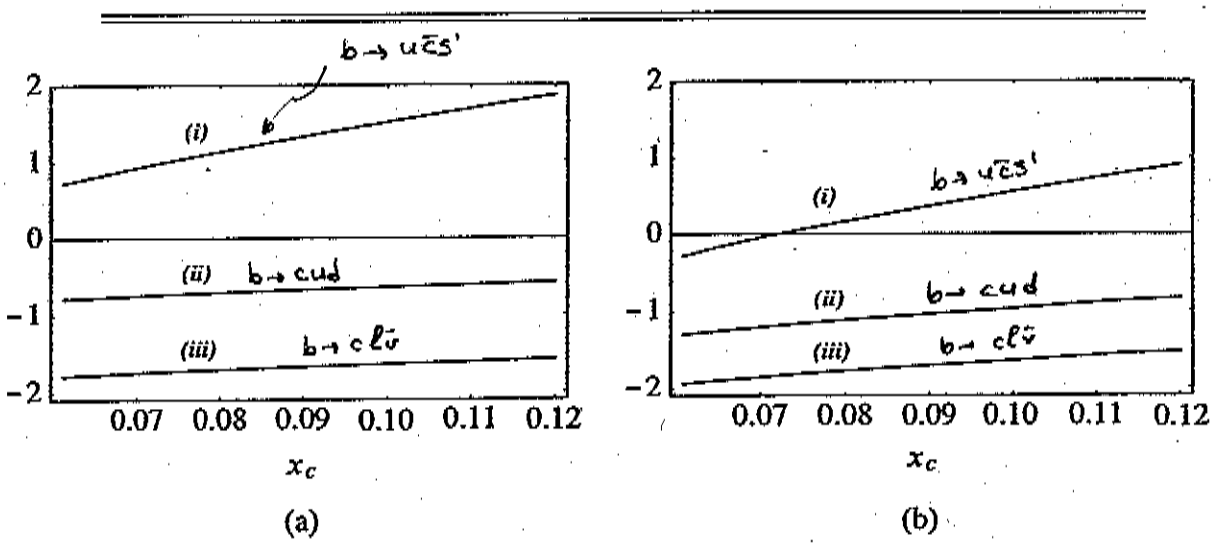


FIG. 2. (a) The one-loop coefficient a_1^X and (b) the BLM-enhanced two-loop coefficient a_2^X to (*i*) $b \to u\bar{c}s$, (*ii*) $b \to c\bar{u}d$ and (*iii*) $b \to cl\bar{\nu}$ decays.

- Estimate the size of the $\alpha_s^2(m_b)$ term

$$R_1 = \chi \left|\frac{V_{ub}}{V_{cb}}\right|^2 \left\{1 + g_1(x_c)\frac{\alpha_s(m_b)}{\pi} + f_1(x_c)\beta_0 \left(\frac{\alpha_s(m_b)}{\pi}\right)^2 + \dots\right\},$$

$$\delta_2 = \left|\frac{V_{ub}}{V_{cb}}\right|^2 \left\{1 + g_2(x_c)\frac{\alpha_s(m_b)}{\pi} + f_2(x_c)\beta_0 \left(\frac{\alpha_s(m_b)}{\pi}\right)^2 + \dots\right\}.$$

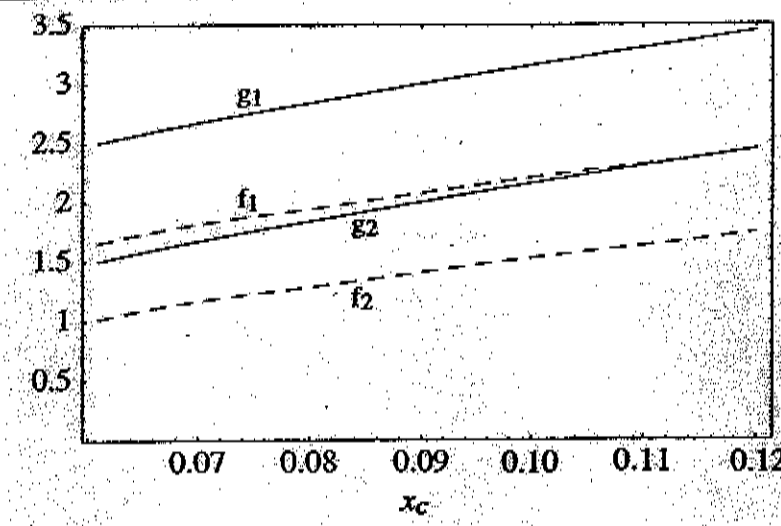
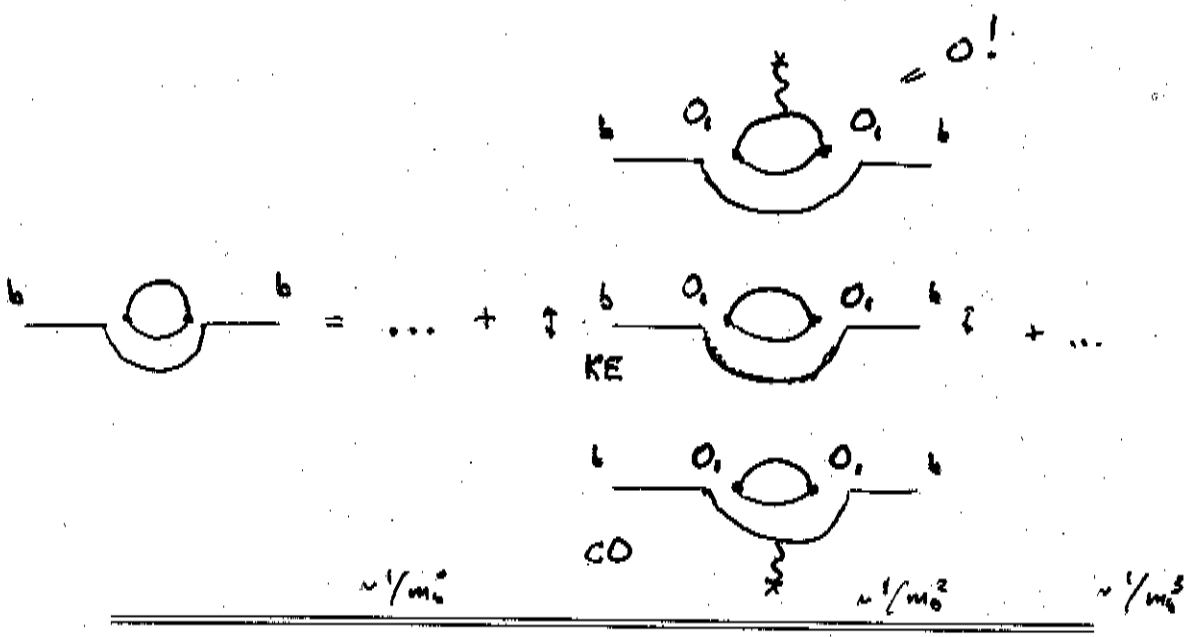


FIG. 3. The one-loop coefficients g_i (solid) and BLM-enhanced two-loop coefficients f_i (dashed).

$$r_1(\mu = m_b, x_c = 0.09) \;\rightarrow\; r_1 + 1.09 \times f_1(0.09) = 0.40 + 0.10 = 0.50\,,$$
$$r_2(\mu = m_b, x_c = 0.09) \;\rightarrow\; r_2 + f_2(0.09) = 0.12 + 0.06 = 0.18\,.$$

Non-perturbative QCD corrections

- Leading m_b dependence cancels out in R_i
- Leading $1/m_b^2$ corrections cancel out in R_i



- ... just like in semileptonic decay: A. Falk et.al. L. Koyrakh
  $b \to ql\bar{\nu}$ is symmetric as $m_q \leftrightarrow m_l$ at $1/m_b^2$

1. ...phase space function is symmetric
2. ...R_i involve ratios of total decay widths
   ($\Pi^{\bar{c}s}_{\mu\nu}(q^2)$ is symmetric as $m_c \leftrightarrow m_s$)

(also, I. Bigi et.al.)

Non-perturbative QCD corrections: $\alpha_s \lambda_2 / m_b^2$

- Two sources:
  1. pQCD corrections to $1/m_b^2$ terms (small...)
  2. terms $\sim C_1 C_2$: $\alpha_s \log(M_W/m_b)\lambda_2/m_b^2$ I. Bigi et.al

$$
\begin{aligned}
R_1 &= |V_{ub}/V_{cb}|^2 \{1 + \ell_2(\mu, x_c) + \ldots\}, \\
\delta_2 &= |V_{ub}/V_{cb}|^2 \{1 + 2\ell_2(\mu, x_c) + \ldots\},
\end{aligned}
$$

where

$$
\ell_2(\mu, x_c) = C_1(\mu) C_2(\mu) \frac{16(1-x_c)^3}{f(x_c)} \frac{\lambda_2(\mu)}{m_b^2}
$$

$$
\lambda_2(\mu) = \lambda_2(m_b)[\alpha_s(\mu)/\alpha_s(m_b)]^{9/25}.
$$

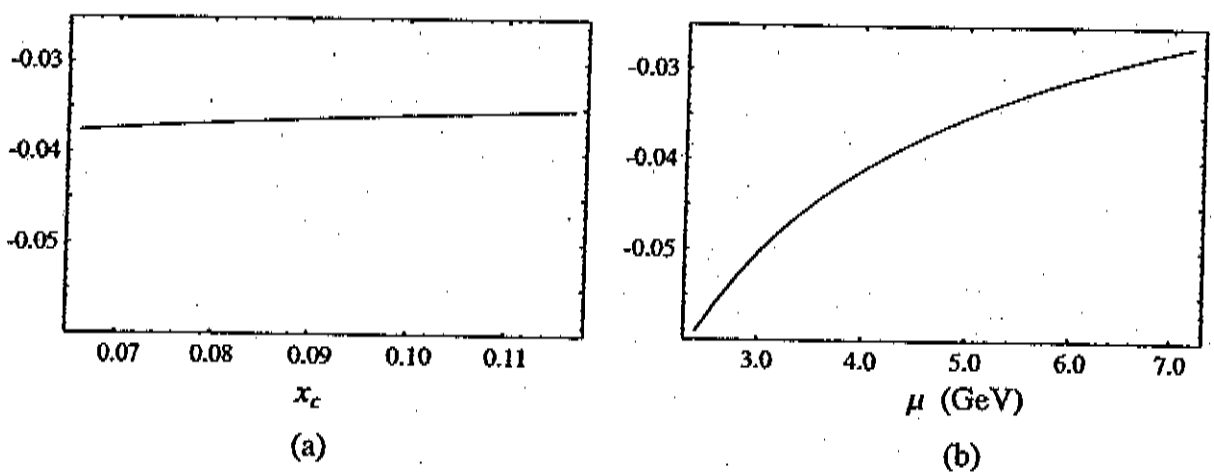
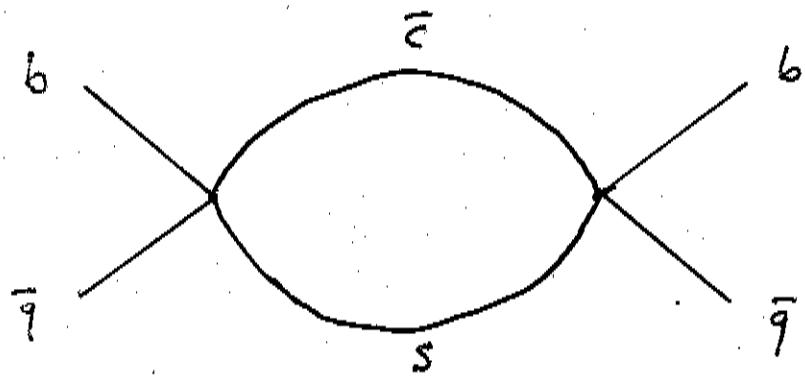


FIG. 4. Variation of ℓ_2 with x_c and μ. (a) $\ell_2(x_c)$ for $4.5\,\text{GeV} \leq m_b \leq 5.1\,\text{GeV}$ and $\mu = m_b$. (b) $\ell_2(\mu)$ for $2.4\,\text{GeV} \leq \mu \leq 7.2\,\text{GeV}$ and $m_b = 4.8\,\text{GeV}$.

Leading non-perturbative corrections

- Leading $1/m_b^3$ corrections (weak annihilation) Phase space ($16\pi^2$) enhancement!



$$
\begin{aligned}
\mathcal{T}_{\rm sp} \quad &= \frac{G_F^2 m_b}{3\pi} |V_{ub}|^2 (1-x_c)^2 \{ N_c (c_1 + c_2/N_c)^2 \\
&\qquad \times \left[(1+2x_c) O^u_{S-P} - (1+x_c/2) O^u_{V-A} \right] \\
&\quad + 2c_2^2 \left[(1+2x_c) T^u_{S-P} - (1+x_c/2) T^u_{V-A} \right] \\
&\quad + \sin^2\theta_C \, N_c (c_2 + c_1/N_c)^2 \\
&\qquad \times \left[(1+2x_c) O^d_{S-P} - (1+x_c/2) O^d_{V-A} \right] \\
&\quad + 2\sin^2\theta_C \, c_1^2 \left[(1+2x_c) T^d_{S-P} - (1+x_c/2) T^d_{V-A} \right] \},
\end{aligned}
$$

where we define

$$
\begin{aligned}
O^q_{V-A} &= \bar{b}_L \gamma^\mu q_L \, \bar{q}_L \gamma_\mu b_L \,, \\
O^q_{S-P} &= \bar{b}_R q_L \, \bar{q}_L b_R \,, \\
T^q_{V-A} &= \bar{b}_L \gamma^\mu T^a q_L \, \bar{q}_L \gamma_\mu T^a b_L \,, \\
T^q_{S-P} &= \bar{b}_R T^a q_L \, \bar{q}_L T^a b_R \,.
\end{aligned}
$$

- Spectator-dependent corrections!

- Parameterize matrix elements

$$
\begin{aligned}
\langle B_q | O^q_{V-A} | B_q \rangle &= \tfrac{1}{4} f^2_{B_q} m^2_{B_q} B_1 \,, \\
\langle B_q | O^q_{S-P} | B_q \rangle &= \tfrac{1}{4} f^2_{B_q} m^2_{B_q} B_2 \,, \\
\\
\langle B_q | T^q_{V-A} | B_q \rangle &= \tfrac{1}{4} f^2_{B_q} m^2_{B_q} \epsilon_1 \,, \\
\langle B_q | T^q_{S-P} | B_q \rangle &= \tfrac{1}{4} f^2_{B_q} m^2_{B_q} \epsilon_2 \,.
\end{aligned}
$$

- Expectations

  Vacuum insertion: $B_1 = B_2 = 1$, $\epsilon_1 = \epsilon_2 = 0$

  In general: $B_1 \sim B_2 \sim 1$, $\epsilon_1 \sim \epsilon_2 \sim 1/N_c$

- Fractional contribution of $1/m_b^3$ terms

$$
\begin{aligned}
R_1 &= |V_{ub}/V_{cb}|^2 \{1 + a_1(B^-, \overline{B}^0) + \ldots\} \,, \\
\\
\delta_2 &= |V_{ub}/V_{cb}|^2 \{1 + a_2(B^-, \overline{B}^0) + \ldots\} \,,
\end{aligned}
$$

- Calculating them we find

$$
\begin{aligned}
a_1(B^-) &= \frac{16\pi^2 f_B^2(1-x_c)^2}{\chi N_c m_b^2 f(x_c)} \{N_c(\overbrace{c_1 + c_2/N_c}^{\text{small ...}})^2 \\
&\quad \times [(1+2x_c)B_2 - (1+x_c/2)B_1] \\
&\quad + 2c_2^2 [(1+2x_c)\epsilon_2 - (1+x_c/2)\epsilon_1] \},
\end{aligned}
$$

$$
\begin{aligned}
a_1(\overline{B}^0) &= \frac{16\pi^2 f_B^2(1-x_c)^2}{\chi N_c m_b^2 f(x_c)} \underline{\sin^2\theta_c} \{N_c(c_2 + c_1/N_c)^2 \\
&\quad \times [(1+2x_c)B_2 - (1+x_c/2)B_1] \\
&\quad + 2c_1^2 [(1+2x_c)\epsilon_2 - (1+x_c/2)\epsilon_1] \},
\end{aligned}
$$

set $N_c = 3$

$$
\begin{aligned}
a_2(B^-) &= \frac{16\pi^2 f_B^2(1-x_c)^2}{m_b^2 f(x_c)} \{(C_1 + \tfrac{1}{3}C_2)^2 \\
&\quad \times [(1+2x_c)B_2 - (1+\tfrac{1}{2}x_c)B_1] \\
&\quad + \tfrac{2}{3}C_2^2[(1+2x_c)\epsilon_2 - (1+\tfrac{1}{2}x_c)\epsilon_1] \\
&\quad - \tfrac{1}{3}(C_1^2 + 6C_1C_2 + C_2^2)B_1 - 2(C_1^2 + C_2^2)\epsilon_1\},
\end{aligned}
$$

$$
\begin{aligned}
a_2(\overline{B}^0) &= -\frac{16\pi^2 f_B^2(1-x_c)^2}{m_b^2 f(x_c)} \underline{\cos 2\theta_C} \{(C_2 + \tfrac{1}{3}C_1)^2 \\
&\quad \times [(1+2x_c)B_2 - (1+\tfrac{1}{2}x_c)B_1] \\
&\quad + \tfrac{2}{3}C_1^2[(1+2x_c)\epsilon_2 - (1+\tfrac{1}{2}x_c)\epsilon_1]\}
\end{aligned}
$$

- ... or numerically

$$
\begin{aligned}
a_1(B^-) &= -0.48B_1 + 0.54B_2 - 0.018\epsilon_1 + 0.021\epsilon_2\,, \\
a_1(\overline{B}^0) &= -0.00034B_1 + 0.00038B_2 - 0.018\epsilon_1 + 0.020\epsilon_2\,,
\end{aligned}
$$

$$
\begin{aligned}
a_2(B^-) &= -0.43B_1 + 0.54B_2 - 1.14\epsilon_1 + 0.021\epsilon_2\,, \\
a_2(\overline{B}^0) &= 0.0063B_1 - 0.0071B_2 + 0.34\epsilon_1 - 0.38\epsilon_2\,.
\end{aligned}
$$

- Employing "vacuum insertion" ansatz

$$
\begin{aligned}
a_1(B^-) &= 0.062\,, & a_1(\overline{B}^0) &= 4.4 \times 10^{-5}\,, \\
a_2(B^-) &= 0.112\,, & a_2(\overline{B}^0) &= -8.1 \times 10^{-4}\,.
\end{aligned}
$$

- Not really a reliable estimate...

$\Rightarrow$ Extract B_i and ϵ_i from lifetime measurements ?

$\Rightarrow$ Lattice, QCD sum rules, ... ?

Survey of results for B_i and ϵ_i

- Lattice (UKQCD, 1998):

$$B_1(m_b) = 1.06 \pm 0.08\,, \qquad \epsilon_1(m_b) = -0.01 \pm 0.03\,, \leftarrow$$
$$B_2(m_b) = 1.01 \pm 0.06\,, \qquad \epsilon_2(m_b) = -0.02 \pm 0.02\,,$$

- QCD sum rules (Chernyak, 1995)

$$B_1 \simeq 1\,, \qquad \epsilon_1 \simeq -0.15\,, \leftarrow$$
$$B_2 \simeq 1\,, \qquad \epsilon_2 \simeq 0\,,$$

- QCD sum rules (Baek *et. al*, 1998)

$$B_1(m_b) = 1.01 \pm 0.01\,, \qquad \epsilon_1(m_b) = -0.08 \pm 0.02\,, \leftarrow$$
$$B_2(m_b) = 0.99 \pm 0.01\,, \qquad \epsilon_2(m_b) = -0.01 \pm 0.03\,,$$

- HQET QCD sum rules (Cheng and Yang, 1999)

$$B_1(m_b) = 0.96 \pm 0.04\,, \qquad \epsilon_1(m_b) = -0.14 \pm 0.01\,, \leftarrow$$
$$B_2(m_b) = 0.95 \pm 0.02\,, \qquad \epsilon_2(m_b) = -0.08 \pm 0.01\,.$$

$\Rightarrow$ ◇ Take CV from LQCD, blow up errors to reflect QCDSR

$$B_1(m_b) = 1.06 \pm 0.10\,, \qquad \epsilon_1(m_b) = -0.01 \pm 0.10\,,$$
$$B_2(m_b) = 1.01 \pm 0.10\,, \qquad \epsilon_2(m_b) = -0.02 \pm 0.10\,,$$

- Our estimate

$$a_1(B^-) = 0.04 \pm 0.07\,, \qquad a_1(\overline{B}^0) = 0 \pm 0.003\,,$$
$$a_2(B^-) = 0.10 \pm 0.13\,, \qquad a_2(\overline{B}^0) = 0 \pm 0.05\,.$$

Murky business of theoretical uncertanties

- Summarizing all sources of uncertanties,

pQCD $\quad \frac{\alpha_s \lambda_2}{m_b^2} \quad$ WA

$$
\begin{aligned}
R_1^- &= |V_{ub}/V_{cb}|^2 \left[1 + r_1(x_c,\mu) + \ell_2(x_c,\mu) + a_1(B^-)\right], \\
R_1^0 &= |V_{ub}/V_{cb}|^2 \left[1 + r_1(x_c,\mu) + \ell_2(x_c,\mu) + a_1(\overline{B}^0)\right], \\
\delta_2^- &= |V_{ub}/V_{cb}|^2 \left[1 + r_2(x_c,\mu) + 2\ell_2(x_c,\mu) + a_2(B^-)\right], \\
\delta_2^0 &= |V_{ub}/V_{cb}|^2 \left[1 + r_2(x_c,\mu) + 2\ell_2(x_c,\mu) + a_2(\overline{B}^0)\right],
\end{aligned}
$$

- ... or numerically

NLO $\quad$ BLM $\quad \frac{\alpha_s \lambda_2}{m_b^2} \quad$ WA

$$
\begin{aligned}
R_1^- &= |V_{ub}/V_{cb}|^2 \left[1.50^{+0.10+0.005+0.010}_{-0.05-0.025-0.015} \pm 0.07\right], \\
R_1^0 &= |V_{ub}/V_{cb}|^2 \left[1.46^{+0.10+0.005+0.010}_{-0.05-0.025-0.015} \pm 0.005\right], \\
\delta_2^- &= |V_{ub}/V_{cb}|^2 \left[1.21 \pm 0.03 \pm 0.015^{+0.015}_{-0.030} \pm 0.13\right], \\
\delta_2^0 &= |V_{ub}/V_{cb}|^2 \left[1.11 \pm 0.03 \pm 0.015^{+0.015}_{-0.030} \pm 0.05\right].
\end{aligned}
$$

pQCD

WA

- Account for correlations in pQCD errors, add non-perturbative errors in quadrature

$$
R_1^- = |V_{ub}/V_{cb}|^2 \left[1.50 \pm 0.15\right], \quad \delta_2^- = |V_{ub}/V_{cb}|^2 \left[1.21 \pm 0.15\right]
$$

$$
R_1^0 = |V_{ub}/V_{cb}|^2 \left[1.46 \pm 0.10\right], \quad \delta_2^0 = |V_{ub}/V_{cb}|^2 \left[1.11 \pm 0.10\right]
$$

Conclusions and Outlook

◇ A new, theoretically clean method of determination of the CKM parameter V_{ub} is proposed.

◇ The proposed ratios,

$$R_1 = \frac{\Gamma(b \to \bar{c}us')}{N_c\Gamma(b \to cl\nu)} \simeq \chi \left|\frac{V_{ub}}{V_{cb}}\right|^2 (1 + g(x_c) + ...)$$

$$R_2 = \frac{\Gamma(b \to cX) - \Gamma(b \to \bar{c}X)}{\Gamma(b \to c\bar{u}d')} = 1 - \frac{\Gamma(b \to \bar{c}us')}{\Gamma(b \to c\bar{u}d')}$$

$$R_3 = \frac{\Gamma(b \to cX) - \Gamma(b \to \bar{c}X)}{N_c\Gamma(b \to cl\nu)} = \frac{\Gamma(b \to c\bar{u}d')}{N_c\Gamma(b \to cl\nu)} - R_1$$

are free of leading renormalon ambiguities and leading $1/m_b^2$ corrections.

◇ Perturbative QCD α_s corrections enhance R_i by $\sim 25\%$.

◇ Non-perturbative $1/m_b^3$ corrections are $\leq 6\%$.

◇ CAN predict R_i with an accuracy of better than 10%!